Write your name here Surname	Other n	ames
Pearson Edexcel GCE	Centre Number	Candidate Number
Statistics		
Monday 27 June 2016 –	Morning	Paper Reference
Monday 27 June 2016 – Time: 1 hour 30 minute	•	Paper Reference 6684/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them. Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy. **Information**
- The total mark for this paper is 75.
- The marks for each question are shown in brackets

 use this as a guide as to how much time to spend on each question.

 Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. A student is investigating the numbers of cherries in a *Rays* fruit cake. A random sample of *Rays* fruit cakes is taken and the results are shown in the table below.

Number of cherries	0	1	2	3	4	5	≥6
Frequency	24	37	21	12	4	2	0

- (a) Calculate the mean and the variance of these data.
- (b) Explain why the results in part (a) suggest that a Poisson distribution may be a suitable model for the number of cherries in a *Rays* fruit cake.

(1)

(3)

The number of cherries in a Rays fruit cake follows a Poisson distribution with mean 1.5.

A *Rays* fruit cake is to be selected at random. Find the probability that it contains

(c) (i) exactly 2 cherries,

(ii) at least 1 cherry.

Rays fruit cakes are sold in packets of 5.

(*d*) Show that the probability that there are more than 10 cherries, in total, in a randomly selected packet of *Rays* fruit cakes, is 0.1378 correct to 4 decimal places.

(3)

(4)

- Twelve packets of *Rays* fruit cakes are selected at random.
- (e) Find the probability that exactly 3 packets contain more than 10 cherries.

(3)

(Total 14 marks)

2. In a region of the UK, 5% of people have red hair. In a random sample of size *n*, taken from this region, the expected number of people with red hair is 3.

(*a*) Calculate the value of *n*.

3.

	(2)
A random sample of 20 people is taken from this region.	
Find the probability that	
(b) (i) exactly 4 of these people have red hair,	
(ii) at least 4 of these people have red hair.	(5)
Patrick claims that <i>Reddman</i> people have a probability greater than 5% of having In a random sample of 50 <i>Reddman</i> people, 4 of them have red hair.	ng red hair.
(c) Stating your hypotheses clearly, test Patrick's claim. Use a 1% level of signific	cance. (5)
(Total	12 marks)
The random variable R has a continuous uniform distribution over the interval [5, 9	9].
(a) Specify fully the probability density function of R .	(1)
	(1)
	(1) (1)
(b) Find P($7 < R < 10$).	
 (<i>a</i>) Specify fully the probability density function of <i>R</i>. (<i>b</i>) Find P(7 < <i>R</i> < 10). The random variable <i>A</i> is the area of a circle radius <i>R</i> cm. (<i>c</i>) Find E(<i>A</i>). 	

(Total 6 marks)

4. A continuous random variable X has cumulative distribution function F(x) given by

$$F(x) = k(ax + bx^{2} - x^{3}) \begin{cases} x < 2 \\ 2 \le x \le 3 \\ 1 \end{cases}$$

Given that the mode of X is $\frac{8}{3}$.

- (*a*) show that b = 8,
- (*b*) find the value of *k*.

(6)

(4)

(Total 10 marks)

5. In a large school, 20% of students own a touch screen laptop. A random sample of n students is chosen from the school. Using a normal approximation, the probability that more than 55 of these n students own a touch screen laptop is 0.0401 correct to 3 significant figures.

Find the value of *n*.

(Total 8 marks)

6. A bag contains a large number of counters with one of the numbers 4, 6 or 8 written on each of them in the ratio 5:3:2 respectively.

A random sample of 2 counters is taken from the bag.

(*a*) List all the possible samples of size 2 that can be taken.

(2)

The random variable *M* represents the mean value of the 2 counters.

Given that
$$P(M = 4) = \frac{1}{4}$$
 and $P(M = 8) = \frac{1}{25}$,

(*b*) find the sampling distribution for *M*.

(5)

A sample of n sets of 2 counters is taken. The random variable Y represents the number of these n sets that have a mean of 8.

(c) Calculate the minimum value of *n* such that $P(Y \ge 1) > 0.9$.

(3)

(Total 10 marks)

7. The weight, *X* kg, of staples in a bin full of paper has probability density function

$$f(x) = \begin{cases} \frac{9x - 3x^2}{10} & 0 \le x < 2\\ 0 & \text{otherwise} \end{cases}$$

Use integration to find

(a) E(X), (4)

(b) Var(X), (4)

(c)
$$P(X > 1.5)$$
. (3)

Peter raises money by collecting paper and selling it for recycling. A bin full of paper is sold for £50 but if the weight of the staples exceeds 1.5 kg it sells for £25.

(d) Find the expected amount of money Peter raises per bin full of paper.

(2)

Peter could remove all the staples before the paper is sold but the time taken to remove the staples means that Peter will have 20% fewer bins full of paper to sell.

(e) Decide whether or not Peter should remove all the staples before selling the bins full of paper. Give a reason for your answer.

(2)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

June 2016 6684 Statistics S2 Mark Scheme

Question Number	Scheme					
Note : if a otherwise.		working award full marks unless the marksche	me says			
1(a)	Mean = 1.41	B1: Cao Allow 141/100	B1			
	Variance = $\frac{343}{100} - 1.41^2$	M1: using $\frac{\sum fx^2}{100}$ – (their mean) ² or $\frac{100}{99} \left(\frac{\sum fx^2}{100} - (\text{their mean})^2 \right)$ oe NB Allow the square root of this for the M mark. If no working shown for $\sum fx^2$ then you must see 343, 3.43 or a correct answer	M1			
	= 1.4419 (s ² = 1.456)	A1: awrt 1.44 or 1.46 for s^2	A1			
			(3)			
(b)	The mean is close to the variance	B1: Cao - allow alternative wording Allow mean equals variance	B1 (1)			
(c) (i)	$X \sim Po(1.5)$ P(X = 2) = $\frac{e^{-1.5}1.5^2}{2!}$	M1: writing or using $\frac{e^{-\lambda}\lambda^2}{2!} \text{ or } P(X \le 2) - P(X \le 1)$	M1			
	= 0.2510	A1: awrt 0.251	A1			
(ii)	$P(X \ge 1) = 1 - P(X = 0)$ = 1 - e ^{-1.5} 0r 1 - 0.2231 = 0.77686	M1: writing or using $1 - P(X=0)$ oe	M1			
	= 0.77686	A1: awrt 0.777	A1 (4)			
(d)	<i>Y</i> ~Po(7.5)	B1: Writing Po(7.5)	B1			
		M1: writing $P(Y \ge 11)$ or $1 - P(Y \le 10)$ oe	M1			
	$\frac{P(Y \ge 11) = 1 - P(Y \le 10)}{= 1 - 0.8622}$ $= 0.1378 *$	A1: Seeing 1 – 0.8622 leading to 0.1378 cso (both B1 and M1awarded)	A1cso (3)			
(e)	<i>A</i> ~ B(12, 0.1378)	M1: using $(p)^n(1-p)^{12-n}$ where $p = 0.1378$ or 0.138 condone missing <i>n</i> C <i>r</i>	M1			
	$P(A = 3) = {\binom{12}{3}} (0.1378)^3 (0.8622)^9$	M1: $\binom{12}{3}(p)^3(1-p)^9$, with $0 Allow 220 or 12 C 3 instead of \binom{12}{3}$	M1			
	= 0.1516	A1: awrt 0.152	A1 (3)			
			Total 14			

Question Number	Scheme				
2(a)	0.05n = 3	M1: using 0.05 <i>n</i>	M1		
	n = 60	A1: cao NB: for 60 with no incorrect working award M1A1	A1 (2		
(b)	$R \sim B(20, 0.05)$	B1: using or writing B(20, 0.05) in (i) or (ii)	B1		
(i)	$P(R = 4) = {}^{20}C_4 (0.05)^4 (0.95)^{16} OR$ $P(R = 4) = P(R \le 4) - P(R \le 3)$	M1 writing or using P($R \le 4$) – P($R \le 3$) or using ${}^{20}C_4(p)^4(1-p)^{16}$	M1		
	= 0.9974 - 0.9841				
(ii)	$= 0.0133$ $P(R \ge 4) = 1 - P(R \le 3)$ $= 1 - 0.9841$	A1: awrt 0.0133 M1: writing or using $1 - P(R \le 3)$	A1 M1		
	= 0.0159	A1: awrt 0.0159	A1 (5)		
(c)	$H_0: p = 0.05$ $H_1: p > 0.05$	B1: Both hypotheses correct and labelled H ₀ and H ₁ , must use <i>p</i> or π Do not allow p(<i>x</i>)	B1		
	$P(R \ge 4) = 1 - P(R \le 3)$	M1: Writing or using B(50,0.05) AND writing or using $1 - P(R \le 3)$ or $P(R \le 3) = 0.7604$ on its own or one of the following 4 statements leading to a CR. $P(R \ge 7) = 0.0118$ $P(R \le 6) = 0.9882$ $P(R \ge 8) = 0.0032$ $P(R \le 7) = 0.9968$ May be implied by correct CR. Allow any letter	M1		
	$= 0.2396$ CR $R \ge 8$	A1: awrt 0.240 or 0.24 or $R \ge 8$ oe Or 0.7604	A1		
	Insufficient evidence to reject H ₀ , Not Significant. Accept H ₀ 4 does not lie in the Critical region.	M1: dependent on the previous M being awarded. A correct statement – do not allow contradictory non contextual statements. Follow through their Probability/CR and H ₁ . If no H ₁ seen then M0. Ignore their comparison in all cases Then mentally compare their probability as follows: For prob < 0.5 statement must be correct compared to 0.01 for 1 tail test and 0.005 for 2 tailed test. For prob > 0.5 statement must be correct compared to 0.99 for 1 tail test and 0.995 for 2 tailed test. NB :If there is no non-contextual statement given you may award the M1 for a correct contextual statement	M1d		
	No evidence to support <u>Patrick's</u> claim. Or no evidence that people in <i>Reddman</i> have a	A1: cso fully correct solution and correct contextual statement containing the word Patrick if writing about the claim	Alcso		
	probability greater than 5% of having <u>red</u> hair	Or red hair if full context	(5		
			Total 1		

Question Number	Scher	ne	Marks
3(a)	$f(r) = \begin{cases} \frac{1}{4} & 5 \le r \le 9\\ 0 & \text{otherwise} \end{cases}$	B1: Allow $r < 5$ and $r > 9$ instead of 0 otherwise Allow < instead of \leq signs. Any letter may be used - condone mixed letters Must have $f(r)$ - condone $F(r)$	B1 (1)
(b)	$P(7 < R < 10) = 2 \times \frac{1}{4}$ = $\frac{1}{2}$	B1: oe	B1 (1)
(c)	$[E(A) = E(\pi R^{2})]$ E(R ²) = Var(R) + [E(R)] ² or $\int_{5}^{9} \frac{r^{2}}{4} dr$	M1: Using correct formula for $E(R^2)$. This may be in any order or written in words	M1
	E(R) = 7, Var (R) = $\frac{4}{3}$ or $\left[\frac{r^3}{12}\right]_5^9$	B1: Var $(R) = \frac{4}{3}$ or awrt 1.33 and $E(R) = 7$ or $\left[\frac{r^3}{12}\right]_5^9$. These may be implied by a correct answer	B1
	$=50\frac{1}{3}$	A1: Allow awrt 50.3	A1
	E(A) = $50\frac{1}{3}\pi$ oe NB If both E(R) ² and [E(R)] ² are both worked o A marks. The best they can get is M1 B1 A1A0	A1: Allow exact multiple of $\pi \text{ eg } 50.3 \pi$ or awrt 158 Do Not allow 50.3 π ut and neither is selected they lose the final	A1
			(4) Total 6

Question Number		Scheme	Marks
	Mark (a) and (b) together – a	llow a missing k throughout	
4(a)	$f(x) = ak + 2bkx - 3kx^2$	M1: Attempting to differentiate $F(x)$ at least one $x^n \rightarrow x^{n-1}$ M1d: Attempting to differentiate $f(x)$ at	M1
	$\left[\frac{\mathrm{df}(x)}{\mathrm{d}x}\right] = \frac{1}{2kb} - 6kx$	least one $x^n \rightarrow x^{n-1}$. Dependent on previous M mark being awarded. A1: Condone missing $\frac{df(x)}{dx}$	M1dA1
	2kb - 6kx = 0 k (2b - 6x) = 0 2b - 6x = 0	M1d: Putting 2^{nd} differential = 0 Dependent on previous Method mark being awarded	M1d
	$2b - 6 \times \frac{8}{3} = 0$	being awarded M1d: Subst $x = \frac{8}{3}$. Allow with k in. Dependent on previous Method mark being awarded	M1d
	<i>b</i> = 8*	A1: Answer given so must have been awarded all previous marks with no errors	A1 cso
	Alternative method – completing the square		(6)
	$-3k\left(x^2-\frac{2bx}{3}-\frac{a}{3}\right)$	M1: factorising by taking -3 <i>k</i> out	M1
	$-3k\left(\left(x-\frac{b}{3}\right)^2-\left(\frac{b}{3}\right)^2-\frac{a}{3}\right)$ or quoting $\frac{-b}{2a}$	M1: Attempting to complete the square dependent on previous M mark being awarded. $\left(x - \frac{b}{3}\right)^2 \pm c$	M1d
	$-3k\left(x-\frac{b}{3}\right)^2 + \frac{b^2k}{3} + ak$	A1: Correct completed square form	A1
	Max at $x = \frac{b}{3}$	M1d: Selecting their $b/3$ Dependent on previous Method mark being awarded	M1d
	$\frac{b}{3} = \frac{8}{3}$	M1: Putting their $\frac{b}{3} = \frac{8}{3}$. Dependent on previous Method mark being awarded	M1d
	<i>b</i> = 8*	A1: Answer given all steps must have shown all the required steps	A1 cso
(b)	F(2) = 0 eg $k(2a+32-8) = 0$ Or $k(2a+4b-8) = 0$ oe	M1: Attempting to form an equation using $F(2) = 0$, or $F(3) = 1$ or $F(3) - F(2) = 1$. Need to subst in the <i>x</i> value and equate A1: -12 - may be implied by $k = 1/9$. Do	M1
	a = -12 F(3) = 1 eg $k(-36+72-27) = 1$ k(-36+9b-27) = 1 oe	not award if the M1 is not given M1: Forming an equation using two of F(2) = 0 or $F(3) = 1$ or $F(3) - F(2) = 1$	A1 M1
	$k = \frac{1}{9}$	A1: Allow equivalent fractions or awrt 0.111	A1
NB If you s	see $k = 1/9$ award full marks. You may award marks in p		(4)
	a quoted and not proved do not award the A r		Total 1

Question Number	Scheme		Marks
5.	N(0.2 <i>n</i> , 0.16 <i>n</i>)	B1: Mean = $0.2n$ and Var = $0.16n$ oe this may be awarded if they appear in the standardisation as $0.2n$ and either $0.16n$ or $\sqrt{0.16n}$	B1
	$P\left(Z > \frac{55.5 - 0.2n}{\sqrt{0.16n}}\right) = 0.0401$	M1: Using a continuity correction either 55.5 or 54.5	M1
	$\frac{55.5 - 0.2n}{\sqrt{0.16n}} = 1.75$	B1: Using a $z = awrt \pm 1.75$ M1: Standardising using either 55.5, 54.5 or 55 and equal to a z value. Follow through their mean and variance. If they have not given the mean and Var earlier then they must be correct A1: A correct equation. May be awarded for $\frac{55.5-0.2n}{\sqrt{0.16n}} = 1.75$ Condone use of an inequality sign rather than an equals sign	B1M1A1
	$0.2n + 0.7\sqrt{n} - 55.5 = 0$	M1d: This is dependent on the previous method mark being awarded. Using either the quadratic formula or completing the square or factorising or any correct method to solve their 3 term equation. If they write the formula down then allow a slip. If no formula written down then it must be correct for their equation. May be implied by correct answer or $\sqrt{n} = 15$ or 342.25 NB you may award this mark if they use 54.5 for awrt 14.9, -18.4, 221 or 337 55 for awrt -18.4, 14.9,223 or -117 If the answer is not one of these then the method for solving their 3 term equation must be seen.	M1 d
	$\sqrt{n} = 15$	A1: Allow 15 or -18.5 do not need to see <i>n</i> or \sqrt{n} . Condone $n = 15$ or n = -18.5	A1
	n = 225	A1 : cao 225 do not need to see <i>n</i> or \sqrt{n}	A1 (8)
	Alternative method for last 3 marks $(0.2n-55.5)^2 = (-0.7\sqrt{n})^2$ $0.04n^2 - 22.69n + 3080.25 = 0$ n = 225 or $1369/4n = 225$	M1 solving 3 term quadratic in <i>n</i> as above A1 either 225 or 1369/4 or 342.25 A1must select 225	Total 8

Question Number		Sc	heme						Mar	·ks
6.(a)	NB 64 is th	0, 48, 00, 08, 88			B1: At least 4 different pairs (ignore incorrect extras)B1: 6 different pairs with no incorrect extras			B1B1	1 (2)	
(b)	<u> </u>	4		$\frac{6}{\frac{3}{10} \times \frac{3}{10} + \frac{1}{2}}$	$\times \frac{1}{5} \times 2$		8		B1 B1 M1 M1A	.1
	$P(\overline{X} = \overline{x})$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{29}{100}$		$\frac{3}{25}$	$\frac{1}{25}$			
			o extras or o				1			
	B1: Writing	g or using	g P($X = 4$)	$=\frac{1}{2}, P(X =$	$=6)=\frac{3}{10}$	and $P(X = X)$	$(8) = \frac{1}{5}$ Ma	y be seen in(a)	
	M1: A corr M1: A corr	rect meth	od for one od for two	of P(5), P(6) of P(5), P(6)	or P(7) or P(7)	may be may be	implied by implied by	y correct answ y correct answ	wer	
	A1: fully co	orrect tab	ole/list -need	4,5,6,7, 8 a	nd their a	associated p	robabilitie	2S		(5)
(c)	$1 - \left(\frac{24}{25}\right)^n > 0.9$ or $\left(\frac{24}{25}\right)^n < 0.1$ oe			M1: $1 - \left(\frac{24}{25}\right)^n > 0.9$ or $\left(\frac{24}{25}\right)^n < 0.1$ oe seen or used may use = or \leq instead of $<$ = or \geq instead of $>$ Do Not award $\left(\frac{24}{25}\right)^n > 0.1$ oe			M1			
	n > 56.4 A1: Ignore any $n >, n <, n = etc.$ Award if you see awrt 56.4 may be implied by $n = 57$					A1				
	n = 57				A1: ca allow <i>i</i> alterna	o $n = 57$ or	< 57. Do r are given.		A1	
	Alternative 50 0.87 51 0.86 52 0.88 53 0.88 54 0.89 56 0.89 57 0.90 58 0.90	0.13 5 0.12 0.12 5 0.11 0.11 4 0.10 8 0.10 2 0.09 6 0.09	Allow Allow awrt 5 66 72 78 74		shown	least 2 trials with correct least $n = 56$	t probabili	ities	M1 A 1	
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0.09			A1: ca allow <i>i</i>		57 on its o < 57. Do r	wn. Do not not award if	A1	(3)
									Tot	al 10

7(a)			
	$\int_{0}^{2} \frac{9x^{2}}{10} - \frac{3x^{3}}{10} dx = \left[\frac{3x^{3}}{10} - \frac{3x^{4}}{40}\right]_{0}^{2}$	M1: using $\int xf(x)$ and attempting to integrate. At least one $x^n \to x^{n+1}$. Ignore limits A1: Correct integration - Ignore limits	M1A1
	$=\left(\frac{3\times2^3}{10}-\frac{3\times2^4}{40}\right)$	M1d: substituting correct limits -dependent on previous Method mark being awarded	M1d
	= 1.2	A1: 1.2 oe. Allow 1.20	A1 (4)
(b)	$E(X^2) = \int_0^2 \frac{9x^3}{10} - \frac{3x^4}{10} dx$	M1 using $\int x^2 f(x)$ and attempting to	
	$=\left[\frac{9x^4}{40} - \frac{3x^5}{50}\right]_0^2$	integrate. At least one $x^n \rightarrow x^{n+1}$. Ignore limits	M1
	$=\frac{42}{25}=1.68$	A1: Allow equivalent fractions. May be implied by a correct answer. Condone Var(X) = 1.68 M1d: use of $E(X^2) - E(X)^2$	A1
	$Var(X) = 1.68 - 1.2^2$		M1d
	= 0.24	A1: cao allow 0.240 or 6/250e	A1 (4)
	$[P(X > 1.5) =] \int_{1.5}^{2} \frac{9x}{10} - \frac{3x^2}{10} dx \text{or} 1 - \int_{0}^{1.5} \frac{9x}{10} - \frac{3x^2}{10} dx$	M1: writing or using $\int_{1.5}^{2} \frac{9x}{10} - \frac{3x^2}{10} dx$ or $1 - \int_{0}^{1.5} \frac{9x}{10} - \frac{3x^2}{10} dx$ Must have correct limits or using $1 - F(1.5)$ for this distribution	M1
	$= \left[\frac{9x^2}{20} - \frac{3x^3}{30}\right]_{1.5}^2 \text{ or } 1 - \left[\frac{9x^2}{20} - \frac{3x^3}{30}\right]_0^{1.5}$	A1 Correct Integration. Condone missing 1-	A1
	$=\frac{13}{40}=0.325$	A1cso: 0.325 or 13/40 oe	Alcso
NB	Watch out for using 1 – f(1.5) or $1 - \frac{9(1.5) - 3}{10}$	$\frac{B(1.5)^2}{2}$. This gets M0A0A0	(3)
	$(0.325) \times 25 + (1 - 0.325) \times 50 = \text{\pounds}41.875$	M1(<i>their</i> (c))×25+(1- <i>their</i> (c))×50 Allow use of their part (c) or 0.325 ie they may restart. Allow 50 – (part(c))×25 A1: awrt 41.9	M1A1 (2)
(e)	$\pounds 50 \times 0.8$ or $\pounds 40$ or 0.4 or awrt 0.038 or awrt 0.163 Peter should not remove the staples as the expected amount earned per bin will be less.	M1:Allow $(50 \times 0.8)n$ or £40 $n (n \neq 0)$ NB Allow 20% off (of) 50 =£40 A1ft: Correct statement containing the word staples and one of the 4 comparisons (ft on (c) or (d)) or the difference in these values must be seen. £40 n < part(d)× n or 0.4 < their part (c) or 0.6 < 1-their part(c)	M1 A1ft (2)
_		or awrt 0.838 > 0.8 or 0.162 < 0.2	Total 15